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On the Determination of Cœlostat Errors.

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(*Communicated by Professor H. H. Turner.*)

§ 1. Introduction.—Professor Turner has already* described the method of determining the errors of a cœlostat by attaching a theodolite to the polar axis, and reading the apparent declinations of stars. By this means we can determine the errors of the mounting in level and azimuth, but not the parallelism of the mirror to the axis. For this latter purpose he either reverses the mirror in its bearings or else follows a star for a sufficient length of time to infer the want of parallelism from the movement of the image in declination. The cœlostat which we employ cannot be tested in the way described by Professor Turner, first because it does not admit of the attachment of a theodolite, and secondly because the mirror is not reversible. These shortcomings are more than made up for by the simplicity of design which results, and we have devised new methods of determining the cœlostat errors which may be useful to those who adopt a similar construction.

§ 2. Authors' First Method.—The easiest way to understand the method we employ is to remember that a cœlostat in perfect adjustment is equivalent to a mercury pool on the terrestrial equator, and the means which we use are the same as those suitable for proving that a mercury pool is on the equator. Now the traveller who wishes to establish this fact observes the maximum or meridian double-altitude of a star. If he is on the equator this double-altitude is twice the star's polar distance, and, similarly, if with the sextant we observe the angle between a bright star and its image in the cœlostat mirror, this angle attains a maximum value of twice the polar distance if the cœlostat is in perfect adjustment. If the cœlostat is out of adjustment so that its normal meets the celestial sphere in a declination Δ, the maximum angle observed is $2(p-\Delta)$, where p is the star's polar distance. Of course the observation of a maximum angle with a sextant is a somewhat tedious process, but we can easily avoid the strain if we elect to observe only at the time of theoretical maximum, for which purpose a watch giving sidereal time to within about one minute is needed. The cœlostat should also be fitted with a rough hour circle. The index of this circle can be made to read 0^h when the cœlostat normal is on the meridian, as follows:—The cœlostat is rotated till the mirror comes into a nearly vertical plane. An Abney's level, reading zero, is pointed at it, and the slow motion of the cœlostat is turned until the image of the Abney level is sighted on its own cross thread. The mirror is then vertical, and the index should be made to read 6^h, after which it will give correct

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hour angles. Let us suppose that we wish to make an observation with α Lyræ at an hour angle of $+3^h$. The right ascension of the star is $18^h\ 33^m$. The cœlostat hour circle is set to $+3^h$, and the observation is made at local sidereal time $21^h\ 33^m$. The observed angle will then be a maximum.

§ 3. Details of Procedure.—At least three stars must be selected for observation, one near the meridian and the other two at distances of at least three hours east and west respectively. The hour circle is set to the correct hour angle, the sextant is clamped at a reading equal to twice the polar distance, and the telescope is pointed at the reflection of the star. The sextant is then slowly rotated round the optic axis in the usual way, and the doubly-reflected star image will be seen entering the field, and at the appointed time the two images are brought to contact. Usually three contacts are observed for each star at intervals of ten minutes, the hour circle being of course moved on ten minutes each time.

§ 4. Calculation of Constants.—It is evident that if we can obtain the value of Δ at at least three points in the track which the cœlostat normal sweeps out as it travels from the east horizon to the west, we can determine the constants of the instrument. For let m and n be the rectangular co-ordinates of the pole of the cœlostat and let γ be the error of parallelism; then evidently $\Delta = m \sin t + n \cos t + \gamma$, and three equations of this form will determine m, n, and γ. It should be remembered that the apparent polar distance of stars is affected by refraction, and the measured distance by errors of graduation of the sextant, and for accurate work these must be allowed for. The former correction is found from the usual formulæ, and the latter is usually found from the certificate of comparison of the sextant. Where none is provided, the sextant can be calibrated by measuring the distances between pairs of bright stars and comparing it with theory.

§ 5. The Observations.—In our observations the refraction is allowed for in each observation, and the sextant (an 8-inch by Troughton & Simms) having no certificate, was calibrated by observing a large number of star-distances. The table which follows gives the results of our observations and also the differences between the experimental values of the difference Δ, and the values computed from the formula $\Delta = m \sin t + n \cos t + \gamma$, where $m = 6'45''$, $n = 16'36''$, and $\gamma = -5'24''$, which numbers were found from the least square solution of the observation equations.

§ 6. Authors' Second Method.—In this method a theodolite with a vertical circle is mounted so that when its telescope is in the meridian it can be pointed at the mirror of the cœlostat. In order that the telescope may move over a wide range while pointing at the mirror, the theodolite is placed as near as possible to the cœlostat.

Now if the cœlostat is accurately adjusted, both it and the telescope can be set so that the light from a star within a certain range of declination is reflected by the mirror along the telescope.

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TABLE I.

Giving Results of Sextant Observations on Cœlostat Errors.

Star.	Observed North Polar Distance.	Correction for Refraction and Sextant Error.	Corrected North Polar Distance (Mean).	Ephemeris North Polar Distance.	Difference.	Hour Angle.	Mean Hour Angle.	Residual on Difference.
	51° 27′ 58″	− 9″ − 15″				$-0^h\,33^m$		
α Lyræ . .	28 43	= −24″	51° 28′ 04″	51° 17′ 46″	+10′ 18″	−0 23	$-0^h\,23^m$	+ 8″
	28 43					−0 13	−5° 45′	
	70° 26′ 42″	+47″ − 30″				+4 29		
Arcturus . .	27 12	= +17″	70° 26′ 59″	70° 20′ 27″	+ 6′ 32″	+4 39	$+4^h\,39^m$	+ 9″
	26 12					+4 49	+69° 45′	
	61° 21′ 15″	+20″ − 25″				−4 04		
α Andromedæ	22 48	= −5″	61° 22′ 15″	61° 24′ 55″	− 2′ 40″	−3 54	$-3^h\,54^m$	+11″
	22 58					−3 44	−58° 30′	
	54° 46′ 37″	+20″ − 0″				−4 34		
β Andromedæ	47 02	=20″	54° 47′ 26″	54° 51′ 57″	− 4′ 31″	−4 24	$-4^h\,24^m$	−18″
	48 38					−4 14	−66° 0′	
	38° 41′ 12″	−10″ − 10″				+3 06		
γ Draconis .	41 17	= −20″	38° 40′ 32″	38° 29 41″	+10′ 51″	+3 16	$+3^h\,16^m$	−15″
	40 07					+3 26	+49° 0′	

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This can still be done if the adjustment of the cœlostat is not perfect, and from the observed dip of the telescope, and the known declination and hour angle of the star, the departure of the cœlostat normal from the celestial equator can be determined.

We shall now give a short account of the method employed and the theory upon which it is based.

§ 7. DETAILS OF PROCEDURE:—

1. The theodolite is mounted to the south of the cœlostat (in the northern hemisphere), and then, by means of a setting on a distant mark whose azimuth is known, the telescope is brought approximately into the meridian.

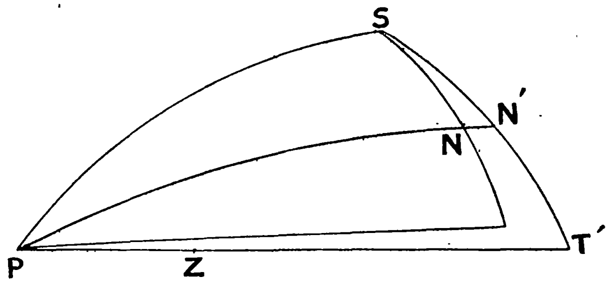
2. The zero of the vertical circle is then determined by a setting on a star at culmination.

3. If the approximate sidereal time is not known, it can be found from direct observation of a star's altitude.

4. A star is selected within the proper range of declination. The cœlostat mirror is then turned so that its normal moves out from the meridian towards the star through half the hour angle of the star. This can be done by means of the hour circle mentioned in the previous method.

The star can now be sighted by setting the telescope to the theoretical dip, when it should be in or near the field of view. The telescope is then set by moving it in the meridian until the image of the star travels along the horizontal crosswire. The sidereal time and the reading of the vertical circle are observed. If this is done for stars with hour angles about 0 and + 4 and − 4 hours, values of the constants giving the departure of the normal can be found, as shown in the preceding part of the paper.

The following is a *résumé* of the theory of the method. The diagram is a projection on a sphere from a centre which is a point on the cœlostat mirror where a ray from the star is reflected along the axis of the telescope.



P is the pole, Z the zenith, T the axis of the telescope, N the normal to the cœlostat mirror, S the star, N′ and T′ are the positions of the normal and telescope when the cœlostat is adjusted correctly, *i.e.* N′ is on the equator of the sphere.

Z T′ is the complement of the theoretical dip of the telescope. Hence P T′ − P T is approximately the difference between the calculated and observed dip. We shall call this d. N N′ is

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the departure of the cœlostat normal from the equator, which we shall call Δ.

It can be shown that $PT' = 180^\circ - PS$
$= 90^\circ + \text{declination of } S$

and that $SPN' = N'PT' = \frac{1}{2}t$ where t is the hour angle of S.

By solving the triangles SPN, NPT, and using the above relations, we find to the first order of small quantities

$$\Delta = \tfrac{1}{2}d \sec \frac{t}{2}.$$

The theoretical dip, d, can be found as follows:

$$\begin{aligned} PT' &= 90^\circ + \text{Decl. of } S = 90^\circ + \delta \\ PZ &= 90^\circ - \text{Latitude of place of observation} \\ &= 90^\circ - \phi. \end{aligned}$$

$$\begin{aligned} \text{Hence } d &= 90^\circ - ZT' = 90^\circ - (PT' - PZ) \\ &= 90^\circ - (\phi + \delta). \end{aligned}$$

From this the telescope can be set to find the star, and the range of declination within which stars can be observed can also be found.

The following example will explain the method:—

TABLE II.

Star's Name.	β Cassiopeiæ.	η Draconis.	α Ursæ Majoris.
Star's hour angle $= 2t$	$0^h\ 18^m\ 52^s$	$8^h\ 35^m\ 56^s$	$-8^h\ 13^m\ 24^s$
Star's true declination $= \delta$	$58^\circ\ 39'\cdot0$	$61^\circ\ 43'\cdot1$	$62^\circ\ 14'\cdot3$
Star's apparent declination $= \delta'$	$58\ 38\cdot4$	$61\ 43\cdot0$	$62\ 14\cdot9$
Theodolite reading	$92\ 21\cdot0$	$88\ 55\cdot0$	$88\ 06\cdot0$
Same corrected $= \zeta'$	$92\ 02\cdot3$	$88\ 36\cdot3$	$87\ 47\cdot3$
$180 - (\phi + \delta')$	$91\ 31\cdot1$	$88\ 26\cdot5$	$87\ 54\cdot6$
$\zeta' - \{180 - (\phi + \delta')\} = d$	$31\cdot2$	$09\cdot8$	$07\cdot4$
Hour angle of cœlostat $= t$	$0^h\ 9^m\ 26^s$	$4^h\ 17^m\ 58^s$	$-4^h\ 6^m\ 57^s$
$\Delta = \frac{1}{2}d \sec t =$	$+15'\cdot6$	$+11'\cdot2$	$-7'\cdot8$

The zero correction $-18'\cdot7$ applicable to the readings of the theodolite was found from a pointing to γ Ursæ Minoris at inferior culmination.

From the observations given in the table, values of Δ are found, and from these the constants giving the deviation of the normal for any hour angle are found from the equation

$$\Delta = m \sin t + n \cos t + \gamma.$$

The values determined by the above experiment are

$$m = 11'\cdot3 \quad n = 24'\cdot8 \quad \gamma = -9'\cdot6$$

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As, in the interval between the observations by the authors' first and second methods, the cœlostat had been dismounted and taken to pieces, no significance attaches to the differences in the values of m, n, and γ given by the two solutions.

§ 8. CONCLUSION.—An examination of the residuals given for Δ in Table I. shows that the first method is a satisfactory one for determining the errors of a cœlostat, and that they can be found with more accuracy than is generally necessary. The observations present no difficulty to one accustomed to the use of the sextant. To those unfamiliar with this instrument, the registering of contacts often presents considerable difficulty, but when we consider how valuable an instrument is the sextant, capable of performing many of the duties of a small theodolite, at a fifth of the cost and the weight, facility in using it is a habit strongly to be encouraged.

The second method can be made as accurate as is required by the use of a good theodolite and the observation of several stars. The instrument used in the above determination was an old 5-inch tacheometer, from which great accuracy could not be expected.

Babbage's Analytical Engine. By Major-General H. P. Babbage.

(Plate 15.)

Charles Babbage died in 1871. He commenced the Difference Engine about 1822, and the Government abandoned it in 1842. He commenced the Analytical Engine about 1833.

A few years before his death he began to execute in metal that part which he called the "Mill," where the operations of addition, subtraction, multiplication, and division can be performed.

I will give here an extract from Part II. of his *Essay towards the Calculus of Functions*, 1816, because it shows the working of his mind at that time, which further on led him to devote his life to the perfection of machinery for the numerical solution of Functions arrived at by mathematical analysis:—

"My subsequent inquiries have produced several new methods of solving functional equations containing only one variable quantity and much more complicated, and have convinced me of the importance of the Calculus, particularly as an instrument of discovery in the more difficult branches of analysis. Nor is it only in the recesses of this abstract science that its advantages will be felt: it is peculiarly adapted to the discovery of those laws of action by which one particle of matter attracts or repels another of the same or a different species; consequently it may be applied to every branch of natural philosophy, where the object is to discover by calculation from the results of experiment the laws which regulate the action of the ultimate particles of bodies. To the accomplishment of these desirable purposes, it must be confessed that it is in its present state unequal; but should the

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labours of future inquirers give to it that perfection which other methods of investigation have attained, it is not too much to hope that its maturer age shall unveil the hidden laws which govern the phenomena of magnetic, electric, or even of chemical action."

To which I would add also the laws which govern our Cosmos, the leading object of this Society.

By his will he left all that existed of his analytical engine, its drawings and notations, as well as his workshop and tools, to me, his youngest son. I was at that time on furlough in England, and after his death I endeavoured to complete the work as far as the instructions in the hands of the workmen admitted, and in that state it remains in the South Kensington Museum at the present day.

I had to return to India at the end of my leave, but retired from the Indian Service in 1874.

Among the things which came to me in his will there was a large number of pieces fitted and ready for the difference engine; but many of the frame plates had been cut up and used by my father for experimental work. I thought to replace them and put together what existed. I began by having plates made to complete the framework as originally designed, and to put in their places the pieces, some completely finished and others less so. I also contrived a new driving gear which answered perfectly, but I found the expenses heavy and the utility doubtful, and about 1879, not knowing what to do with it, I sent the whole (excepting five or six small pieces sent to different places) to the melting-pot.

About that time I heard that Mr. Wilkinson, a nephew and successor of Clement, had broken up his workshop and melted up what remained there of the unfinished pieces of the difference engine. Had I known this in time I might have kept what I had, for I still hold to the opinion expressed in *Babbage's Calculating Engine*, published in 1889, that the calculating part of the difference engine might have been completed, at the time the Government gave it up, for, say, £500.

In 1856 I turned my attention to the machine of the Messrs. Schentz, and, with a view of instructing myself, I applied my father's system of Mechanical Notation to their machine. My father was very pleased with my work, and sent me to exhibit my diagrams at the meeting of the British Association at Glasgow. The diagrams are now in the South Kensington Museum, and I have employed the notation for the present machine, and have found it very useful.

Soon after my return to England I began to think that I might be able to complete the "Mill," not as part of the larger machine which my father proposed, but something which would be practically useful by itself in the hands of a skilled operator.

The iron top and bottom frame plates were cast in 1880, and the work was carried on, somewhat intermittently, until the 21st. of January 1888, when the machine produced a table from 1st. to 44th. multiples of π to twenty-nine places of figures, which could

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have been prolonged indefinitely; but the machinery did not work smoothly, and in the 32nd. multiple of π the carriage had failed and a figure had gone wrong. For a time I was discouraged, and in 1896 circumstances led me to discontinue the work, and I turned to other less expensive pursuits and offered the unfinished work of my father, with all the drawings and papers belonging to it, as well as my own imperfect work, to the South Kensington Museum, and the offer was accepted.

In 1906 my attention was accidentally attracted to my work. I saw what was amiss and the way to remedy it, with the result that it was sent to the factory of Mr. R. W. Munro at Tottenham, and, under my close supervision, it has been completed and the apparatus made to print the calculated results on paper, from which a zinc block has been made.

The printed calculations which follow are specimens of its work.*

Cheltenham:
1910 *April* 8.

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* It will be seen that the "multiples of π" are really multiples of the first 29 digits of π.

[CALCULATIONS.

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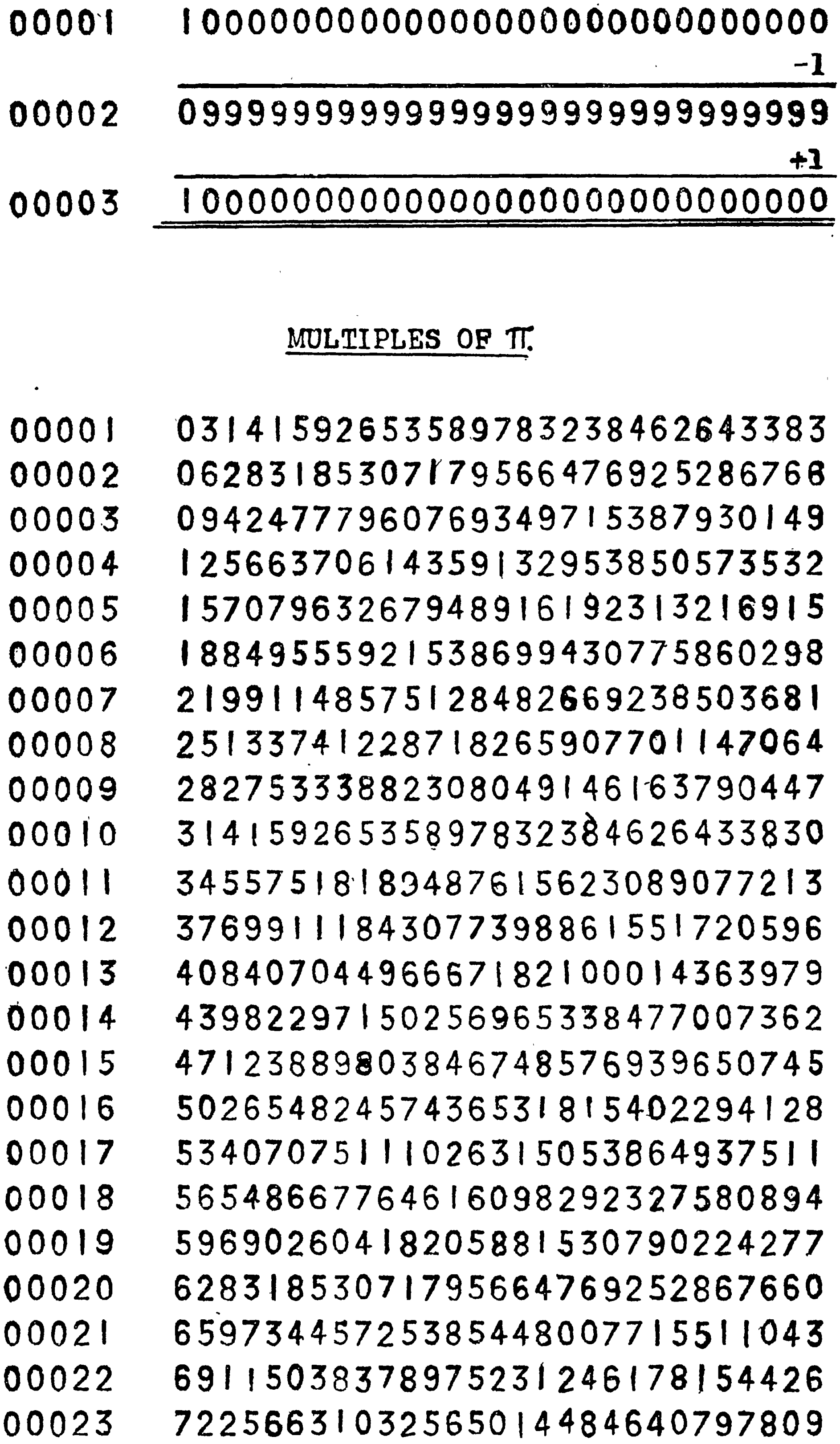

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00001     10000000000000000000000000000
                                     -1
          _____________________________
00002     09999999999999999999999999999
                                     +1
          _____________________________
00003     10000000000000000000000000000
          =============================


               MULTIPLES OF π.

00001     03141592653589783238462643383
00002     06283185307179566476925286766
00003     09424777960769349715387930149
00004     12566370614359132953850573532
00005     15707963267948916192313216915
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00007     21991148575128482669238503681
00008     25133741228718265907701147064
00009     28275333882308049146163790447
00010     31415926535897832384626433830
00011     34557518189487615623089077213
00012     37699111843077398861551720596
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00021     65973445725385448007715511043
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00023     72256631032565014484640797809
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Specimen of work printed by the machine.

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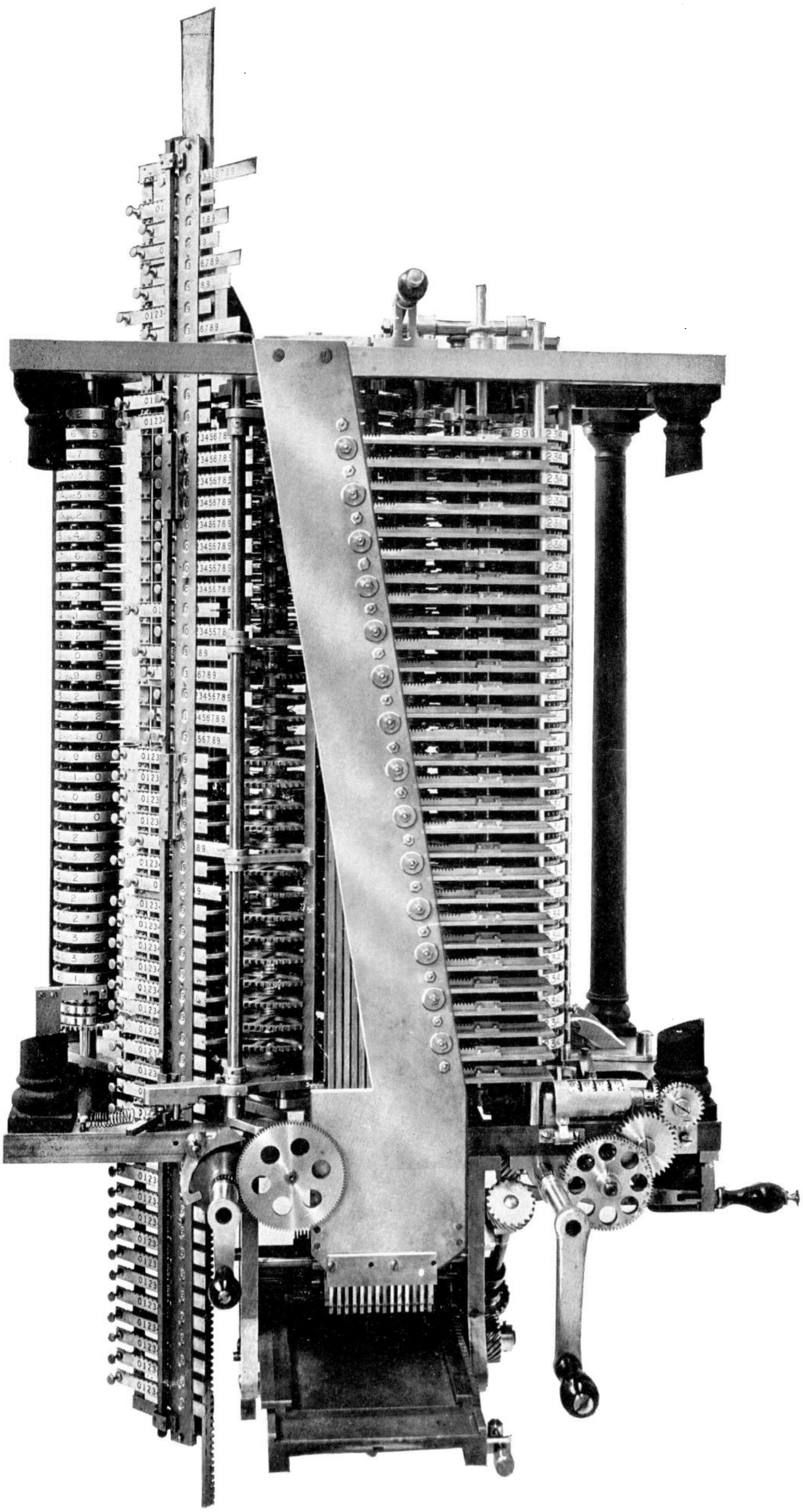

THE MILL AND PRINTING PARTS OF BABBAGE'S ANALYTICAL ENGINE.

Constructed by Major-General H. P. Babbage, 1880–1910.